03 Powers and roots

THE UNIVERSE TROUGHOUT THE POWERS OF TEN

Website reference: http://htwins.net/scale2/

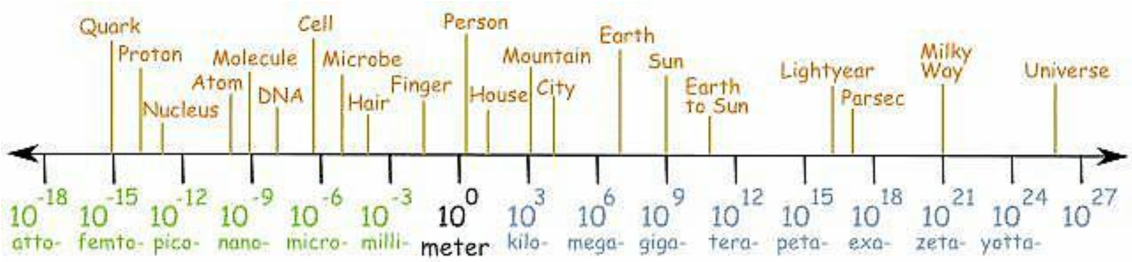
We can travel through the Universe using the powers of ten.

It is interesting to see that the different size not only shows more or less things but different ways to organize the matter.

Could be a man 10 m tall or 100 m tall? Is there a limit to the dimension for a living being?

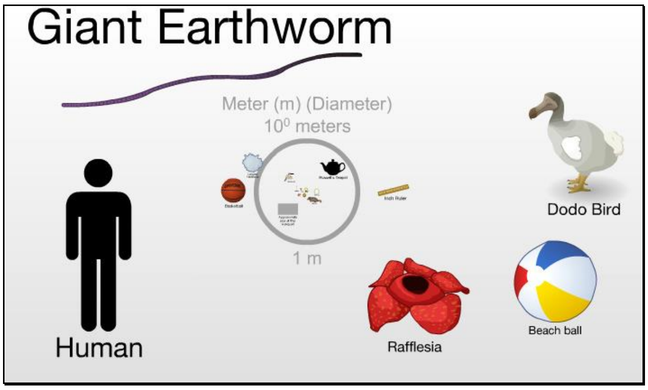
Even the time isn't indifferent. The Universe evolves through the time. You can't find men till a determined point in the development of the Universe.

Here is the entire Universe in a scale according to the size of things. You can check that each order has its proper structure.



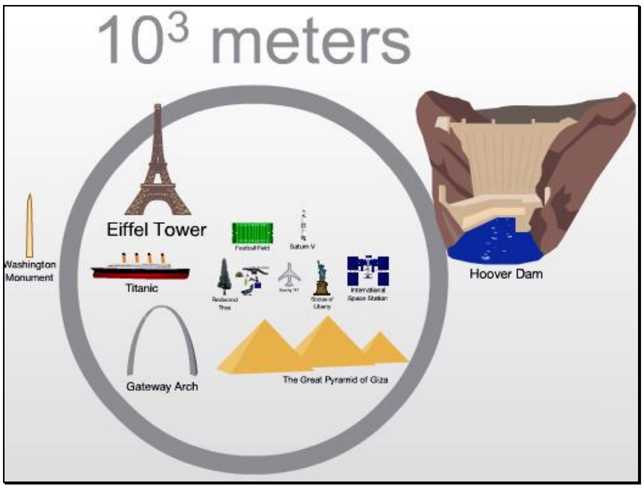
10^0

This is our power, the human dimension. The circle in the picture has 1 m diameter. This is a reference to measure the different objects.



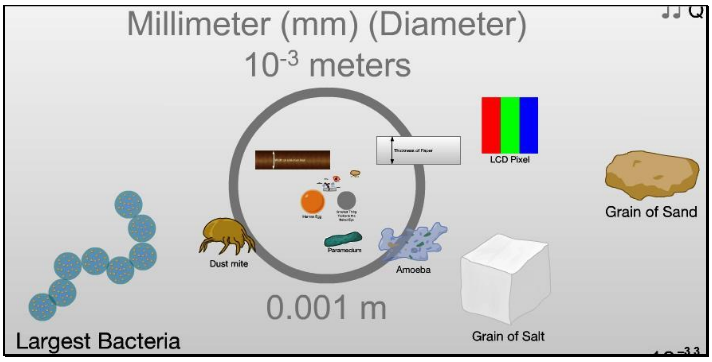
10^3

This is the kilometer, the dimension of a city, the place where we live in. At this scale new objects appear. From this order and upper there aren't living beings.



10^{-3}

This is the millimeter. The order of lowest length we can see to the naked eye.



YOUR TURN

You have to choose three objects, animals or things -one of each order- and write the commentary you can find on the beginning website. You have to explain briefly in class the reason for your election and their features.

Example. Human hair. 100 micrometres. 10^{-4} metres

Human hair is amazing. Straight hair is almost perfectly cylindrical. On the other hand, curly hair is flatter, which allows it to curl, like a ribbon. Did you know that you probably have 50,000 to 200,000 strands of hair in your head? You can count!

1. POWERS WITH INTEGER EXPONENTS

POWERS WITH NATURAL EXPONENTS

This is the product of a number many times by itself. $a^n=b$

a is the base.

n is the exponent.

b is the power. That is the result.

For 3^2 ; we can say:

Three to the power of 2.
Three squared.
Three to the second power.
Three raised to the power 2.

Three raised to the second power.
Three to the second.
Three to two.
For 5^3; we can say also five cubed.

PROPERTIES OF POWERS

Product rule. Product of powers of the same base

To multiply powers of the same base you add their exponents.

$$a^n \cdot a^m = a^{n+m}$$

Example:

$10^3 \cdot 10^5 =$

Quotient rule. Quotient of powers of the same base

To divide powers of the same base we take away the exponents.
(subtract the bottom exponent from the top exponent)

$$a^n : a^m = a^{n-m}$$

Example:

$\frac{10^7}{10^5} =$

Power of a power

When you raise a power to an exponent you multiply those exponents together.

$$\left(a^n\right)^m = a^{n.m}$$

Example:

$\left(10^2\right)^3 =$

Power of a product

When you have a product raised to an exponent raise each factor to that exponent.

$$\left(a \cdot b\right)^n = a^n \cdot b^n$$

Example:

$\left(5x\right)^3 =$

Power of a quotient (a division)

When you have a quotient raised to an exponent raise each number to that exponent.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:

$\left(\frac{x}{2}\right)^3 =$

Exponent 0

Any number to the power of zero is one.

$a^0 = 1$

POWER WITH NEGATIVE EXPONENTS

A number raised to a negative integer is equal to the inverse of the number raised to the positive integer.

$$a^{-n} = \frac{1}{a^n}$$

Example:

$5^{-3} = \frac{5^0}{5^3} = \frac{1}{5^3} = \frac{1}{125}$

Thus, $10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0'001$ and so on.

2. SQUARE ROOT

SQUARE ROOT

The square root of a number 'a' is 'b' means that 'b' squared is 'a'.

$$\sqrt[2]{a} = b \ \ means\ that\ b^2 = a$$

'2' is the index, 'a' is the radicand, 'b' is the radical.

NUMBER OF SQUARE ROOTS OF A VALUE

$\sqrt{0} = 0$. It only has one square root.

$\sqrt{4} = \pm 2$. It has two square roots.

$\sqrt{-9} = it\ doesn't\ exist$. A negative number has no square roots. There is not a number whose square can be -9.

GEOMETRIC MEANING OF A SQUARE ROOT

In a square we have the following relations:

Surface = side^2

Area is the side squared.
In the opposite way:

$$Side = \sqrt{area}$$

The side is the square root of the area.

WHOLE AND DECIMAL ROOTS

They are the roots with a whole result. They are the radicals whose radicand is a perfect square.

For instance, $\sqrt{81}=9$. The square root of 81 is a perfect root because the result is a whole number.

Other numbers have decimal square roots.

PROPERTIES OF A SQUARE ROOT

The square root of a product is equal to the product of the square roots.

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$$

Example:

$$\sqrt{4\cdot 9}=\sqrt{4}\cdot\sqrt{9}$$

The square root of a division is equal to the division of the square roots.

$$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$$

Example

$$\sqrt{\frac{100}{25}}=\frac{\sqrt{100}}{\sqrt{25}}$$

EXTRACT FACTORS OF A SQUARE ROOT

This is a way to simplify a radical. We can take out factors which are perfect squares.

Example:

$\sqrt[2]{48}=$

ORDER OF OPERATIONS

PEDMAS

Parenthesis, powers (and roots), division and multiplications, additions and subtractions.

3. CUBE ROOT

The cube root of a number 'a' is 'b' means that 'b' cubed is 'a'.

$$\sqrt[3]{a}=b \ \ \textit{means that } b^3=a$$

'3' is the index, 'a' is the radicand, 'b' is the radical.

Example:

$\sqrt[3]{8}=$

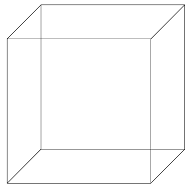
NUMBER OF CUBE ROOTS OF A VALUE

Every number has a cube root only.

GEOMETRIC MEANING OF A CUBE ROOT

In a cube we have the following relations:

Volume = edge^3



Volume is the edge cubed.
In the opposite way:

$$Edge = \sqrt[3]{volume}$$

The edge is the cube root of the volume.

INTEGER AND DECIMAL SQUARES

Perfect cubes have integer cube roots; other numbers have decimal cube roots.

PROPERTIES OF A CUBE ROOT

The cube root of a product is equal to the product of the cube roots.

$$\sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b}$$

The cube root of a division is equal to the division of the cube roots.

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

EXTRACT FACTORS OF A CUBE ROOT

This is a way to simplify a radical. We can take out factors which are perfect cubes.

Example:

$\sqrt[3]{250} =$

EXERCISES AND PROBLEMS

1. Powers with integer exponent

1. Calculate the following expression: a) $\dfrac{5-2\cdot 5^2}{4^2-7}$; b) $\dfrac{10+5\cdot 2^3}{1-2^2}$

2. Calculate: *a*) 5^0; *b*) $(-2)^{-3}$; *c*) $(-2)^5$; *d*) $7'5^2$

3. Calculate: *a*) 5^{-2}; *b*) $(-2)^4$; *c*) $(-1)^5$; *d*) $2'5^2$

4. Write as an unique power the following expression:

a) $10^3\cdot 10^5$ *b*) $10^{-2}\cdot 10^6$ *c*) $\dfrac{10^5}{10^7}$ *d*) $\left(10^2\right)^3$

5. Simplify the following expression as much as possible: $\dfrac{\left(a^2\cdot b^3\right)^2\cdot\left(a\cdot c^4\right)^3}{\left(a^5\cdot b^3\cdot c^2\right)^2\cdot b^3}$

6. Simplify the following expression as much as possible: $\dfrac{\left(a^2\right)^3\cdot\left(c^4\right)^2}{a^5\cdot b^3\cdot c^6}$

7. Write in scientific notation: a) 24 000 000; b) 0'000 45

8. Write in decimal notation the following numbers:

a) 5.10^3 *b*) $8'2.10^{-2}$ *c*) $6'1.10^{-4}$ *d*) $3'67.10^{10}$

9. Write in scientific notation the following data:

a) The number of cells of the human body is about 70 billions.

b) The size of the flu virus is 0'000 000 22 cm

c) The age of the Sun is about 5 thousand millions years.

10. Write in scientific notation the following numbers using only three significant digits:

a) 98 765 430 000 b) 0'000 004 567 3

b) Complete the equality with a suitable exponent in order to get a correct equality; that is, to write in scientific notation correctly:

$a)\ 384.15\cdot 10^5 = 3.8415\cdot 10$

$b)\ 0.003\cdot 10^{-2} = 3\cdot 10$

$c)\ -543.19\cdot 10^{-6} = 5.4319\cdot 10$

11. Simplify giving the final result as a power of ten:

$a)\ 1.2\cdot 10^3\cdot 2\cdot 10^5;\ b)\ 5.6\cdot 10^6\cdot 3\cdot 10^8;\ c)\ -4.2\cdot 10^3\cdot 5\cdot 10^{-7}$

12. They needed 100 000 workers during 40 years to build the Queops pyramid.

Write in scientific notation the expenses by supposing we would have to pay €600 per month to each worker.

13. Answer the following questions: in a) and b) write as an unique power, in c) write as a power whose base is a prime number, in d) write in scientific notation.

a) $(-3)^5:(-3)^{-2}$; *b*) $\left(7^3\right)^2$; *c*) $\dfrac{1}{243}$; *d*) $0'00000003456783$

14. Answer the following questions: in a) and b) write as an unique power, in c) write as a power whose base is a prime number, in d) write in scientific notation.

a) $3^5 : 3^7$; *b*) $\left(3^5\right)^2$; *c*) $\frac{1}{256}$; *d*) 765356000

15. Simplify using the properties of the powers (write 4 and 9 as powers of 2 and 3, respectively, before doing the simplification)

a) $(-2)^3 \cdot 2^4$ *b*) $\left(\frac{2}{3}\right)^5 \cdot \left(\frac{2}{3}\right)^{-3}$ *c*) $2^{-3} \cdot 4^2$ *d*) $3^{-1} \cdot 9^2$

16. Write as a unique power: a) $2^4 \cdot 2 \cdot 8$; b) $4^3 \cdot 4^5 \cdot 16$; c) $\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{-3} \cdot \frac{1}{16}$

17. Write as power of 2 the following numbers:

$8 \quad 16 \quad 8^{-1} \quad 16^{-2} \quad \frac{1}{16} \quad \left(\frac{1}{2}\right)^{-2}$

18. Write as power of 3 the following numbers:

$27 \quad 81 \quad 27^{-1} \quad 81^{-2} \quad \sqrt[3]{9} \quad \frac{1}{27} \quad \left(\frac{1}{3}\right)^{-2} \quad 9^{\frac{1}{2}}$

19. Write as a power whose base is a prime number:

$a)\ -128;\ b)\ \frac{1}{625};\ c)\ \left[\left(2^2\right)^{-3}\right]^4;\ d)\ \frac{1}{125}$

20. Write as a power whose base is a prime number:

$a)\ -343;\ b)\ \frac{1}{64};\ c)\ \left[\left(3^4\right)^{-2}\right]^3;\ d)\ \left(\frac{1}{9}\right)^3$

21. The distance between the Earth and the Sun is about 150 millions km. If you consider the Earth orbit as a circumference, what is its perimeter? And what is the Earth speed through it?

2. Square root

22. Evaluate the following expression step by step: $\left(-3\sqrt[3]{64} - 2^4 : 4 + 3\right) \cdot \sqrt{49}$

23. Evaluate the following expression: $\left(5\sqrt[3]{8} - 2 \cdot 3^2 + 4\right) \cdot \sqrt{100}$

3. Cube root

24. Simplify by taking out of the root the largest factor as much as possible:
$5\sqrt{12} + 7\sqrt{48} - \sqrt{108} - \sqrt{192} + \sqrt{243}$

25. Simplify by taking out of the root the largest factor as much as possible: *a*) $\sqrt{18}$; *b*) $\sqrt{75}$; *c*) $\sqrt[3]{24}$; *d*) $\sqrt[3]{250}$

26. Simplify by taking out of the root the largest factor as much as possible: *a*) $\sqrt{32}$; *b*) $\sqrt{147}$; *c*) $\sqrt[3]{40}$; *d*) $\sqrt[3]{54}$

27. Simplify the following radicals. Explain your results: $a)\ \sqrt{50}$ $\quad b)\ \sqrt[3]{\frac{1}{1000}}$ $\quad c)\ \sqrt[3]{864}$

28. Calculate the following quantities by using the calculator:

a) $\sqrt[3]{48}$ *b*) $\sqrt[4]{150}$ *c*) $\sqrt[3]{835}$ *d*) $\sqrt[3]{689}$

Write the result with three significant digits.

29. Using the calculator do the following operations: $a)\ \sqrt[9]{1.287\cdot 0^{-14}}$ $b)\ 129.8^{15}$

Write the result as a power of ten.